# BOOK REVIEW

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HIGHER MATHEMATICS FOR BEGINNERS AND ITS APPLICATION IN PHYSICS (in Russian), 5-th edition. "Nauka", GL Red. Fiz. -Matem. Lit-ry. (Main Editorial Office for Physico-Mathematical Publications) Moscow, 1970

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This book of 559 pages contains two parts: the first devoted to mathematics and the second to its application to various problems of physics. The main feature of the second part is the exposition of simplified statements of physical problems and of the use of mathematics as a computing tool.

The main impression of the two parts of the book is that of absence of logical lucidity, carelessness, excessive verbosity, and the presence of errors, and all this after five editions of huge circulation and expressions of thanks to 13 persons.

To illustrate this let us examine in detail several individual examples.

Page 17. The concept of the derivative of function $z(t)$ is fundamental. On that page appears (in italics) the following: "The ratio $\Delta z / \Delta t$ tends to a definite limit when $\Delta t$ tends to zero". This statement is clearly not always true. For example, on p. 185 and further on (where functions with discontinuities and other singular points are considered). Appropriate qualifications are not only necessary but would help the "beginner" to understand clearly the meaning of the derivative and of the most important mathematical concept of the limit in general.

It should not be assumed that a deeper examination of these fundamental mathematical concepts is too difficult for beginners, particularly when one compares this with the explanation with the Dirac functions and their derivatives appearing in the part dealing with applications.

Futhermore, no definition is given in the book of the notion of function continuity or that of infinitely small quantities of various orders. Even on the most elementary level one can hardly expect to obtain an isea of "higher mathematics" without these notions.

Page 332. The formulation of Newton's laws is extremely careless. There is no mention of the inertial reference system and no clear statement that Newton's laws are assumed to apply only to rectilinear translational motions of a body. The exposition of fundamental concepts of mechanics is consequently vague.

The extension of equations of motion to the case of a heavy projectile moving along a parabola is unconvincing, since generally speaking, the motion of these is not translational, and the notion of the material point is not mentioned in the book.

This shows how fundamental physical concepts and effects are presented in the form of vaguely defined rules valid only from particular point of view, which are not explained to the reader and, in the context of expounded theories, are false. Undoubtedly (even) able young men will not understand the proposed solutions, while undiscerning readers may assimilate the rules and thus get used to patterns of reasoning which are essentially unsatisfactory.

Pages 335, 337 and 340 present examples of verbosity which in parts is exceedingly elementary and in others unnecessary and too difficult (curvature of curves, infinity of curvature, etc.) for beginners. All lengthy disussions of the shock of bodies in the theory of translational motion of these can only create an illusion of understanding.

The concept of the material point and laws of its motion would have considerably simplified the formulation of problems and ensured correct reasoning.

Page 350. It contains an accumulation of crude errors which show a total lack of competence of the Author in problems of hydrodynamic drag of bodies.

The Author defines the hydrodynamic drag of a body in a stream of fluid by the following formula:

$$F = -k\,S\,\rho\,\frac{v_\infty^2}{2}$$

where $S$ is the cross section area of the body, $\rho$ is the density, and $v_\infty$ the velocity of the oncoming flow.

The importance of specifying the related range of $k$ is well known, but the Author points out the constancy of the coefficient $k$ — which depends only on the shape of the body — for bodies including those of perfectly streamlined form with only one condition for the Reynolds number

$$\mathrm{Re} = \frac{\rho v_\infty R}{\eta} > 100$$

where $\eta$ is the viscosity coefficient and $R$ is a linear dimension which the Author had by evident negligence omitted to define. It is clear that, depending on the definition of $R$ and the shape of the body surface, different intervals with upper and lower limits are admissible for the Re number, within which the coefficient $k$ may be considered approximately constant.

The Author's statement about the constancy of $k$ for a sphere of radius $R$ and any $\infty \geqslant \mathrm{Re} > 100$ is false. It is well known that it is also false in the case of other bodies of smooth shape. The dependence $k$ (Re) in the case of a sphere and bodies of other shapes is considered and discussed in almost all textbooks on hydrodynamics. This dependence which in many instances is affected by the fluid viscosity is of considerable practical importance.

With the increase of the Reynolds number in an unlimited range the coefficient $k$ generally changes considerably.

The assertion that for the specified Reynolds numbers the drag is virtually independent of viscosity is a crude error. For well streamlined bodies the hydrodynamic drag produced by viscous friction actually represents 85% or more of the total drag, and the remaining 15% is due to form drag which depends on pressure distribution on the surface of the body in the absence of body separation and is usually induced by and depends on the fluid viscous properties.

The footnote on p. 350 is simply totally wrong. The Author states: "This formula is valid for a Reynolds number $Rv_\infty\rho/\eta > 100$. The formula in the text implies that in the case of motion of a large body the energy expended on overcoming the medium resistance is not absorbed by friction between layers of fluid but on (imparting) kinetic energy of the fluid (to the fluid) forced to move apart for allowing the body to pass. Derive from this yourself the formula for the force." If the correct procedure is followed in the case of well streamlined bodies in the absence of viscosity, the d'Alembert paradox of zero drag is obtained. According to Kirchhoff the drag produced by the flow of a perfect fluid past a body is nonzero, but none of the "beginners" who might follow the path indicated by the Author would be capable of calculating that force (drag).

Within certain ranges (intervals of Re numbers) it is possible to speak of a weak dependence of coefficient $k$ on the Reynolds number hence, also, on the viscosity coefficient. Such dependence does always exist, and it is wrong to provoke the notion (to imply) that a perturbed fluid flow is virtually independent of viscosity. The imprecise treatment of these very important physical problems is harmful to "beginners".

Page 525. Consider the sentence: "The theories of Lobachevskii, Bolyai and Riemann were silent flashes of lightning preceding the "thunder clap" of the general theory of relativity". Obviously there is not much in this that "beginners" can understand.

Page 526. Arguments about the unsuitability of Cartesian coordinates for defining a curved space are trivial. Elements of two- and multidimensional Riemann spaces can be in fact embedded in multidimensional Euclidean spaces. The history and practice of geometry shows that for a number of reasons it is necessary to express oneself with greater precision in a qualified approach to the essence of a problem.

The Author does not agree with the known Laplace statement that mathematics is a particular kind of mill whose output depends entirely on the material fed into it.

In fact, mathematics not only grinds the input but, also, pays considerable attention to the processing of the latter. The Author's disagreement with Laplace's statement stems from fear that the mill may produce something unexpected. This is an example of the Author's illogical reasoning, since the knowledge of input does not necessarily mean that only expected results will be obtained.

In a popular book for "beginners" extensive mathematical generality is obviously not possible and even unnecessary. This does not mean, however, that clearly false or vague statements can be presented without any qualifications.

The essence of mathematics does not amount to simply prescribing methods for dealing with particular examples, but aims at imparting a clear understanding of the intrinsic properties of the mathematical tool and of all fundamental propositions in the formulation of problems.

Clearness of definitions and logical deductions are integral features of mathematics whose absence is the negation and profanation of mathematics. Although intuition in teaching and mathematical investigations is very important, it does not mean that mathematics is to be presented deliberately incorrectly in books and that simple qualifications which can only lead to a better understanding of its essence are to be omitted.

To avoid any misunderstanding we would stress that we favor simplified formulation and simple exposition of fundamental propositions all of which must be, however, clear and true.

The notion that simplicity and clarity must be obtained at the expense of imprecision and errors is false.

In any case, careless and deliberately vague methods of presentation of already existing highly refined theories are absolutely intolerable in books and lectures. Even more intolerable is a frivolous treatment of fundamentals of science. It is inadmissible to produce muddlers and unwary users of formulas right from the beginning of their introduction to mathematics. The subsequent retraining is more difficult than learning anew.

One is struck by the astonishing absurdity of the logic of the book arrangement. Chapters 1, 2 and 3, in which notions of the derivative and of the integral, and problems related to the maximum and minimum of functions presented with the aid of coordinates and curves, are followed by Chapter 4, where the whole (subject) is again explained "from Adam" by defining the concept of a function, explaining the meaning of the coordinate system and the methods of function representation in the form of curves, and by giving examples of the simplest functions and their respective curves.

The unprofessional style of presenting mathematics and laws of mechanics without any qualification as regards the limits of validity of various statements is a feature of the entire content of the book. This is not the way to develop inquisitiveness of the reader, since he is deprived of the opportunity to obtain a real understanding of the essence of the subject. Worst of all, such style may lead the "beginner" or the nonspecialist to the illusion of understanding.

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## AUTHOR'S REPLY

### ON THE TEACHING OF HIGHER MATHEMATICS AND MY BOOK "HIGHER MATHEMATICS FOR BEGINNERS AND ITS APPLICATION IN PHYSICS"

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It is perhaps for the first time that the subject of teaching mathematics appears on the pages of PMM, and I am glad to take this opportunity for presenting my views on it.

In defining the main purpose of teaching mathematics primary consideration must be given to people who will apply it in practice and not to professional teachers, but the latter must have a decisive influence on the elaboration of teaching methods.

I consider the historical approach as the most important principle which is to be taken into consideration in the broadest formulation of teaching. The student should be led through the stages (of science development) which were passed by humanity (memorization of dates and names is not necessary). In many instances one has to have the courage to renounce clearly at the beginning of a course of lectures the latest, more e fashionable, and more rigorous treatments recently developed.

The second general principle is the realization that understanding and creative assimilation of new concepts occur intuitively and are enhanced by practical applications. The introduction of new concepts by rigorous, formally and logically faultless definitions and proofs is pedagogically unsound. The faultlessness will not be appreciated by a person who only begins to get familiarized with a new branch of science. The importance of strictness in the development of science itself and of reverting, after the first intuitive concenter (stage), to fundamentals from strictly defined positions is not denied.

I consider that theaching of higher mathematics must begin in practice with the introduction of notions of the derivative and of the integral, omitting the theory of limits.

Obviously such approach is not rigorous, since the concepts of the derivative and of the integral are based on some specific passing to limit. It is not without fault, since a passing to limit is not always possible and does not always lead to a definite quantity. Although conscious of all this, I nevertheless consider that at the initial teaching stage attention must be fixed on positive content of the notions of the derivative and of the